

FLOW FIELD DURING FLOW OF AN OVEREXPANDED
SUPERSONIC JET AROUND A CONE

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The results are presented of a calculation, by the method of characteristics, of the flow of a supersonic overexpanded jet around a cone. The results of the calculations are compared with the experimentally measured values of the pressure on the cone.

In some engineering applications, it is of practical interest to establish the flow field formed during flow of an overexpanded axisymmetric supersonic jet around a cone. A special feature of the case considered below is that the flow, as it spreads across the cone, is diverted to a direction perpendicular to the axis of symmetry of the incident jet. The semiangle of the cone was selected so that the shock wave in front of it was of the attached type.

The equations of the characteristics of an ideal adiabatic gas written in Elers–Chushkin variables were used for calculating the flow [1].

During flow of an overexpanded jet around a cone, an attached shock wave KB (Fig. 1) forms in front of it and interacts with the shock wave of the free jet RB, forming a configuration of four shock waves (point B in Fig. 1). One of them is reflected from the surface of the cone and interacts with shock wave BD. An increase of pressure occurs at point C where the shock wave is reflected from the cone. This is seen clearly on the graphs of the pressure distribution over the cone (Fig. 2). The flow behind point A, where the boundary of the free jet changes the inclination to the axis of symmetry as a result of interaction with the shock wave, represents an annular supersonic jet which propagates along the generatrix of the cone. As in the case of an overexpanded axisymmetric jet, the boundary of the annular jet has a barrel-shaped structure, which leads to the occurrence of a suspended shock wave. This shock can be reflected from the surface of the cone regularly or irregularly. A subsonic flow forms behind the irregularly reflected shock.

The flow was calculated continuously from the nozzle exit to that section in the flow passing around the cone where it becomes subsonic. It was assumed that a conical flow was realized near the cone, after the calculation of which we constructed the characteristics of the first family, the parameters of which served as the initial parameters in calculating further flow. To calculate the parameters on the shock waves and their interaction, the relations on the characteristics were supplemented by equations establishing the relation between the parameters in front of, and behind, the shock wave located in a nonuniform flow was determined according to a scheme which is presented in detail in [2]. The conical flow was calculated according to the relations presented in [1].

The flow in the neighborhood of the break of the boundary of the jet by the shock wave AD was calculated as a Prandtl–Meyer flow. In particular, we determined the inclination of the boundary of the annular jet to the axis of the nozzle, the Mach number, and the pressure on it, which are constant for the case of discharge into the stationary medium being considered here. The parameters of the flow between the generatrix of the cone and the boundary of the annular jet were determined by the usual finite-difference procedure. During the calculation we determined the magnitude of the static pressure behind the hanging shock wave. If the curve of this pressure had a minimum, it was assumed that the hanging shock wave is reflected from the surface of the cone irregularly and the triple point on the hanging shock is located at the point of minimum pressure.

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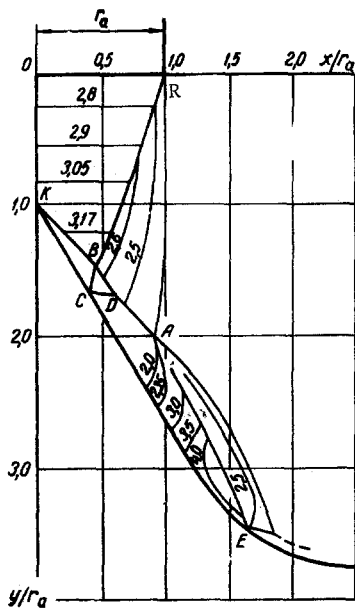


Fig. 1. Geometry of the flow and lines of constant Mach numbers during flow of an overexpanded jet around a cone ($Ma = 2.7$; $k = 1.405$; $\alpha = 7^\circ$; $n = 0.7$).

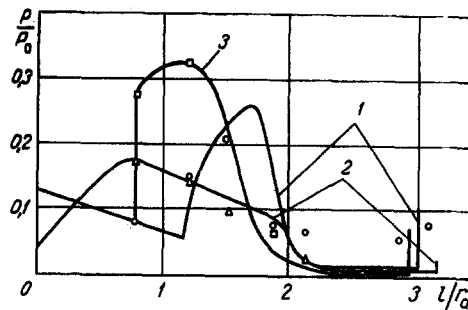


Fig. 2. Pressure distribution over the generatrix of the cone (calculation and experiment for $Ma = 2.7$; $k = 1.405$): 1) $n = 1.0$; $s/r_a = 1.0$; 2) respectively, 1.0 and 3.4; 3) 0.7 and 1.0.

This assumption was used in [3] when calculating axisymmetric jets and its accuracy with respect to experimental results can be considered satisfactory.

The form of the shock wave between the triple configuration and generatrix of the cone is determined after assuming that the inclination of the shock with respect to the sense of the velocity vector in front of it is 90° . The flow in the subsonic region behind this shock was considered further to be one-dimensional, and the position of the slip line leaving the triple point was determined on the basis of the condition of equality of the static pressure on it from the supersonic and subsonic part of the flow.

We will dwell on some results of the calculations. Figure 1 shows the geometry of the flow being formed for one of the calculated variants. The lines of constant Mach numbers are also shown there. We must note first of all that in all calculated variants only one "barrel" formed within which the flow is supersonic. Behind it the flow is subsonic, its thickness being much less than the radius of the nozzle exit. In addition, as a consequence of distortion of the generatrix of the cone near the point of reflection of the hanging shock there is a region in which the Mach number, and so, the pressure are constant.

Figure 2 presents the calculated graphs of the distribution of static pressure on the generatrix of the cone on which are plotted the points obtained experimentally. As we see, their agreement is satisfactory.

NOTATION

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| Ma | is the Mach number at nozzle exit; |
| P_0 | is the stagnation pressure at nozzle exit; |
| P | is the pressure on cone; |
| α | is the semiangle of nozzle; |
| $n = P_a/P_e$ | is the overexpansion of discharge; |
| P_a | is the pressure at nozzle exit; |
| P_e | is the pressure in ambient medium; |
| k | is the adiabatic exponent; |
| s | is the distance from nozzle exit to vertex of cone; |
| l | is the distance reckoned from the vertex of the cone along its generatrix; |
| r_a | is the radius of nozzle exit. |

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